- **Candidates should be able to :**

  - Explain that the **standard (hot big bang) model** of the universe implies a **finite age** for the universe.

  - Select and use the expression : **age of the universe ≈ $1/H_0$**

  - Describe qualitatively the **evolution of the universe 10^{-43} s after the big bang to the present.**

  - Explain that the universe may be **'open', 'flat' or 'closed',** depending on its density.

  - Define the term **critical density.**

  - Select and use the expression for **critical density** of the universe :

$$\rho_0 = \frac{3(H_0)^2}{8\pi G}$$

  - Explain that it is currently believed that the density of the universe is **close to, and possibly exactly equal to,** the critical density needed for a **'flat'** cosmology.

STANDARD (HOT BIG BANG) MODEL OF THE UNIVERSE

- The fact that the universe is expanding not only resolved Olber's paradox, but also suggested when and how, the universe began.

- According to Hubble's law, galaxies are moving apart at a speed which is directly proportional to the distance between them.
  Assuming that the galaxies have always been moving at the speeds we now Observe, there must have been a time, long ago, when all the galaxies in the Universe were in the same place. This suggests that the universe must have started with an immense explosion or big bang.

- This theory is supported by the following observational evidence :

  - The universe is expanding.
  - Hubble's law shows that all galaxies are receding from us.
  - The universe is saturated with cosmic microwave background radiation.
  - The most distant galaxies, and hence the youngest, show a chemical composition of 25% primordial helium.

- The hot big bang model assumes that space and time evolved from a **singularity*** in an event that occurred **about 13.7 billion years ago.**

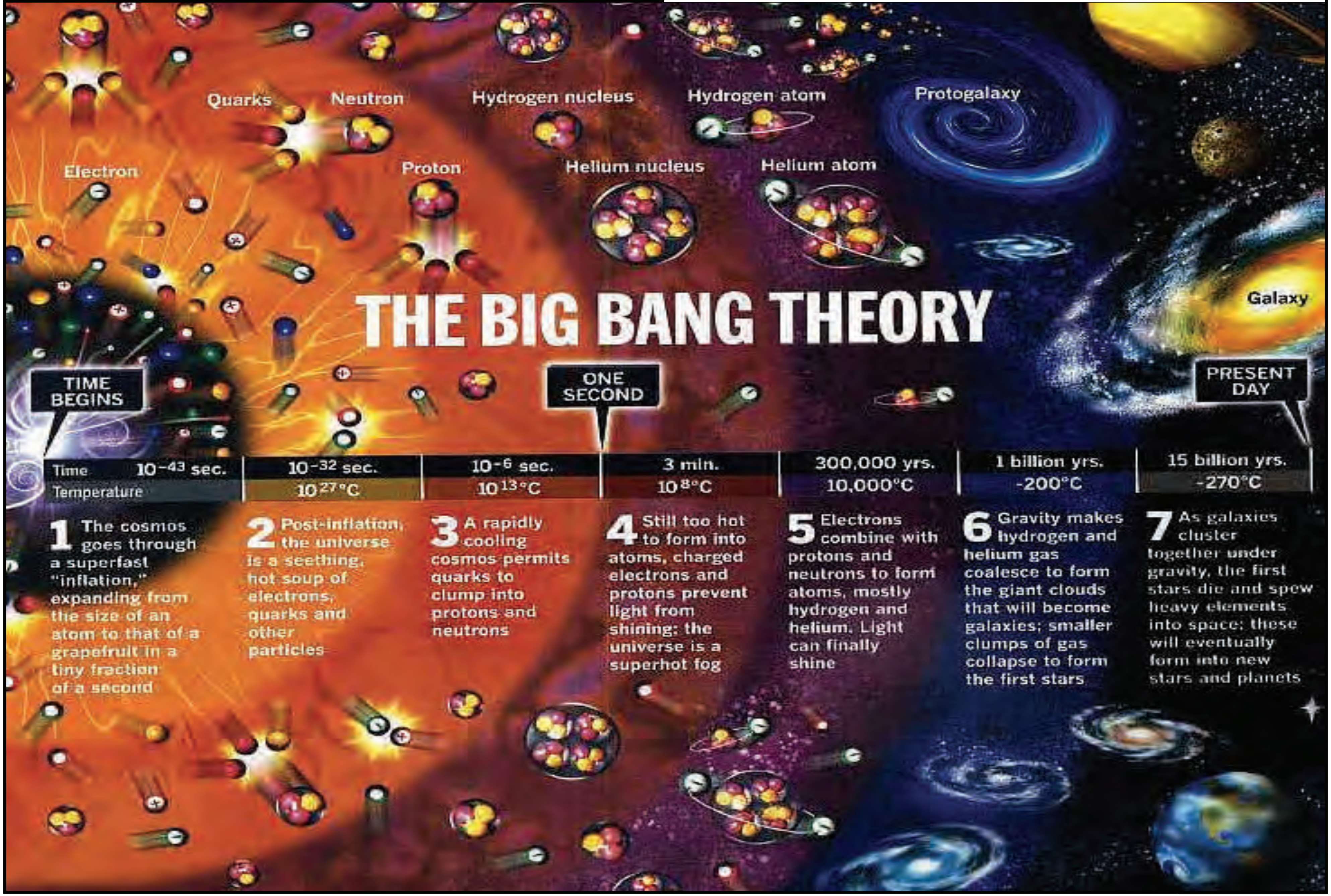
* A **singularity** is a point in space-time in which the laws of physics do not apply. The instant before the big bang occurred is a singularity because all the matter in the universe is thought to have been concentrated at a single point of infinite density.

EVOLUTION OF THE UNIVERSE

- The table below shows the sequence of events from the big bang to the present. The times and temperatures are typical values and apart from ime = 0, they do not represent gradual, rather than abrupt changes.

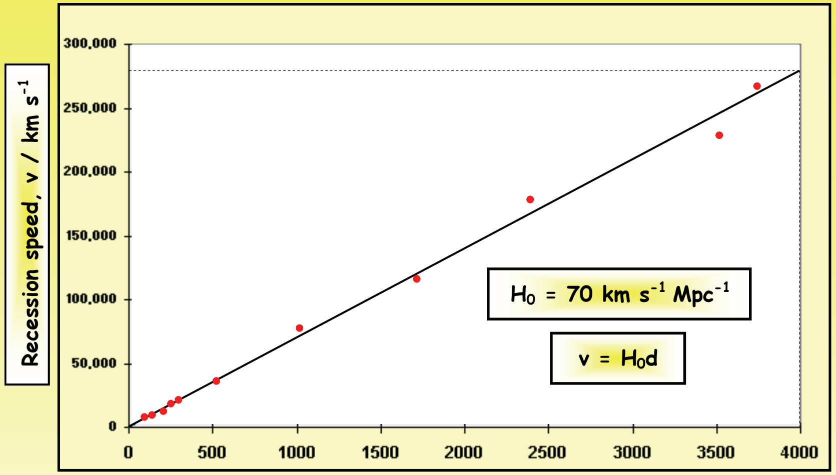
TIME	TEMPERATURE	WHAT WAS HAPPENING
0	infinite	• The universe is **infinitely small, infinitely dense and extremely hot**. • The big bang occurs about **13.7 million** years ago.
10^{-43} s	10^{27} K	• The universe is rapidly expanding and cooling. • All four types of forces (i.e. **gravitational, electromagnetic, strong** and **weak**) are unified. • Size of universe ≈ **10^{-35} cm**.
10^{-34} s	10^{22} K	• The **gravitational** force separates from the other forces. • The universe consists of a primordial soup of **quarks** and **photons**.
10^{-16} s	10^{16} K	• The **strong** force separates from the weak and electromagnetic forces. • The universe is still quite small (size ≈ **2 light minutes**) and for reasons unknown it contains **more matter than antimatter**. • **Leptons** form from **photons**.
10^{-3} s	10^{10} K	• The **weak** and **electromagnetic** forces separate. • The **strong** force becomes dominant and **quarks** are combined to form **hadrons** (including **protons** and **neutrons**). Proton-neutron ratio is about **4 to 1**. • There is much combination of matter and anti-matter until **all antimatter disappears** from the region that becomes the **Milky Way**. • Size of universe ≈ **1 to 2 light years**.

TIME	TEMPERATURE	WHAT WAS HAPPENING
10^{2} s	10^{7} K	• The matter in the universe is in the **plasma** state, consisting of a **soup of free nuclei and electrons**. • Fusion reactions between protons produces a significant amount of **helium** (known as primordial helium). **25%** of the observable universe is helium. • Size of universe ≈ **30 light years**.
10^{5} years	10^{4} K	• The temperature is low enough for **hydrogen and helium atoms** to be formed as electrons become attached to protons. • **Photons travel freely** and the **universe is transparent**. • The **cosmic microwave background (CMB)** radiation is formed.
10^{6} years	6000 K	• The **gravitational** force becomes dominant. • **Hydrogen** and **helium** clumped together to form **stars** and eventually clusters of stars which become **galaxies**. • Size of universe ≈ **1.5 million light years**.
10^{9} years	17 K	• **Heavy elements** formed as a result of the **gravitational collapse of stars**.
13.7 x 10^{9} years	2.7 K	• The **present time**. • There is **life on Earth**. • **Proton-neutron** ratio is still about **4 to 1**. • Present size of the observable universe is estimated at **46.5 x 10^{9} light years!**


THE BIG BANG THEORY
Quarks
Neutron
Hydrogen nucleus
Hydrogen atom
Protogalaxy
Electron
Proton
Helium nucleus
Helium atom
Galaxy
TIME BEGINS
ONE SECOND
PRESENT DAY
Time: 10−43 sec. | 10−32 sec. | 10−6 sec. | 3 min. | 300,000 yrs. | 1 billion yrs. | 15 billion yrs.
Temperature: 1027°C | 1013°C | 108°C | 10,000°C | -200°C | -270°C
1 The cosmos goes through a superfast "inflation," expanding from the size of an atom to that of a grapefruit in a tiny fraction of a second
2 Post-inflation, the universe is a seething, hot soup of electrons, quarks and other particles
3 A rapidly cooling cosmos permits quarks to clump into protons and neutrons
4 Still too hot to form into atoms, charged electrons and protons prevent light from shining; the universe is a superhot fog
5 Electrons combine with protons and neutrons to form atoms, mostly hydrogen and helium. Light can finally shine
6 Gravity makes hydrogen and helium gas coalesce to form the giant clouds that will become galaxies; smaller clumps of gas collapse to form the first stars
7 As galaxies cluster together under gravity, the first stars die and spew heavy elements into space; these will eventually form into new stars and planets

THE AGE OF THE UNIVERSE

- The **Hubble** graph of **recession speed** against **distance from earth** is shown below. It should be noted that the reliability of the **Hubble constant, H_0** (= the gradient of the graph) is poor because the values for distant galaxies are uncertain.



- From the graph it can be seen that a galaxy at a distance **(d)** of **4000 Mpc** has a recession speed **(v)** of **280 000 Km s^{-1}**.

Time taken for the galaxy to reach this position at this speed is given by :

$$\text{Time} = \frac{\text{distance (d)}}{\text{speed (v)}} = \frac{4000 \text{ Mpc} \times 3.09 \times 10^{22} \text{ m Mpc}^{-1}}{2.8 \times 10^{8} \text{ m s}^{-1}}$$

$$= 4.4 \times 10^{17} \text{ s}$$

$$= \frac{4.4 \times 10^{17} \text{ s}}{3.15 \times 10^{7} \text{ s year}^{-1}} = \mathbf{1.4 \times 10^{10} \text{ years} = 14 \text{ billion years}}$$

- This figure is slightly high because the following approximations have been made :

- That the galaxy has been travelling at the **same speed** throughout its existence. This is not true in practice, since it must have **gained some gravitational potential energy and hence lost some of its initial kinetic energy**. This means that the galaxy's **present speed is less than its average speed** and if the latter had been used in the calculation, a lower value would have been obtained for the age of the universe.

- There is an **uncertainty in the value of H_0** and this means an that the value obtained for time (t) is also uncertain.

- The **galaxies formed some considerable time after the big bang** and this time delay has been ignored in the calculation.

For all the above reasons, it is preferable to say that:

Age of the universe ≈ $1/H_0$

CRITICAL DENSITY (ρ_c) OF THE UNIVERSE

- The question cosmologists are asking is this :

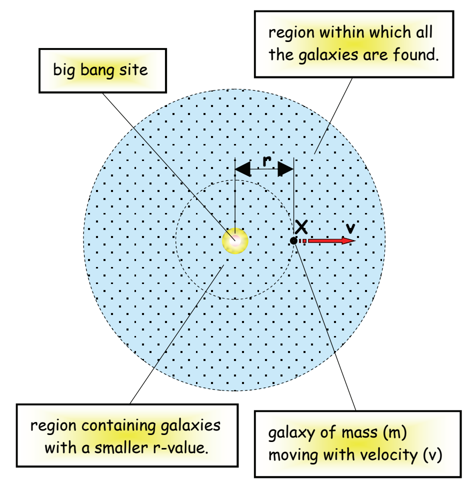
"Have the galaxies got enough kinetic energy to go on and on for ever or will they, at some time in the future, have expended all their kinetic energy and be pulled back to the initial singularity point in a 'big crunch'?"

And the answer to this question depends very much on the **mean density of the universe**.

EQUATION FOR THE CRITICAL DENSITY (ρ_c) OF THE UNIVERSE

- The diagram opposite shows a galaxy **X** of mass **(m)**, moving with velocity **(v)**, at a distance **(r)** from the site of the **big bang**.

  The **total** mass of all the other galaxies between **X** and the big bang site = **M**.



- For **X** to travel outwards for ever, it must overcome the attractive forces of the galaxies which lie behind it, i.e. it needs to gain **gravitational potential energy (Ep)** given by :

$$E_p = GMm/r \qquad (\text{and} \quad M = V\rho = 4/3\,\pi r^3\rho)$$

$$E_p = \frac{G \times 4/3\,\pi r^3\rho \times m}{r} = \frac{4\pi G r^2 \rho m}{3}$$

The **kinetic energy (E_k)** of **X** is given by : $\quad E_k = \frac{1}{2} mv^2$

But, $\quad v = H_0 r \qquad$ So : $\quad E_k = \frac{1}{2} m H_0^2 r^2$

- If $\quad$ **$E_p < E_k$**, $\quad$ X carries on travelling for ever.
- If $\quad$ **$E_p > E_k$**, $\quad$ X will eventually stop and be pulled back.

- The limiting condition is when $\quad$ **$E_p = E_k$** $\quad$ and then :

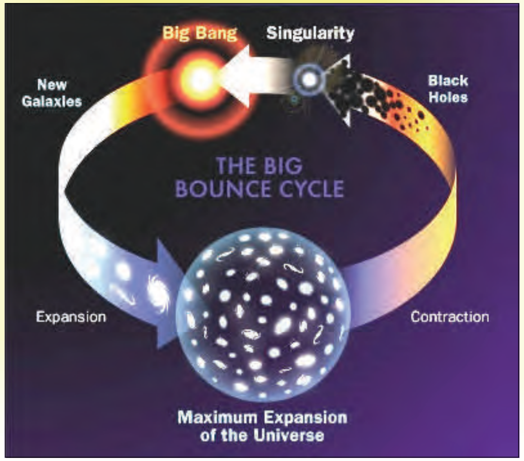
$$\frac{4\pi G r^2 \rho_c m}{3} = \frac{m H_0^2 r^2}{2}$$

From which the **critical density (ρ_c) of the universe** is given by :

$$\boldsymbol{\rho_c = \frac{3H_0^2}{8\pi G}}$$

- If the present density, **ρ** of the universe is **less than** the **critical density, ρ_c, the universe will carry on expanding for ever.**

- If **ρ is greater than ρ_c**, then **the universe will eventually stop expanding and collapse, producing the big crunch.**



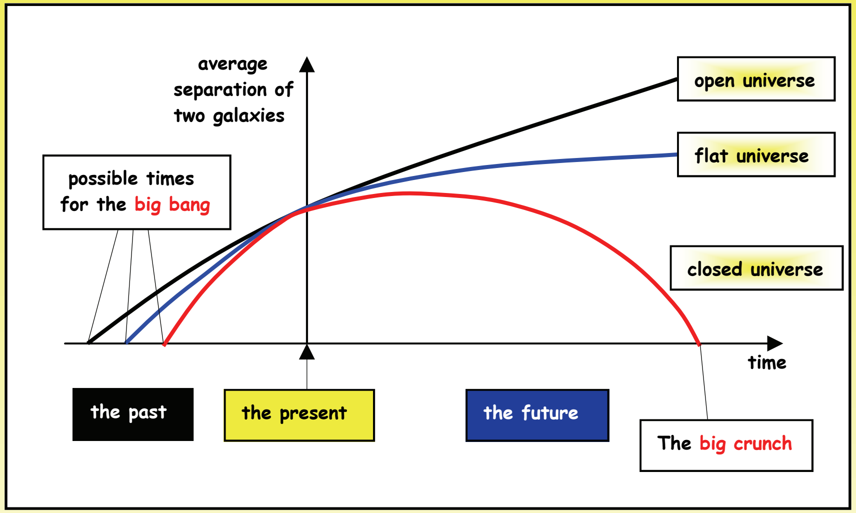
- $$\rho_c = \frac{3H_0^2}{8\pi G} = \frac{3 \times (2.3 \times 10^{-18})^2}{8\pi \times 6.67 \times 10^{-11}} = \boxed{9.5 \times 10^{-27}\ \text{kg m}^{-3}}$$

- Using the cosmological principle that **the universe is homogeneous**, the **mean density, ρ of the universe** is found to be ≈ **10^{-27} kg m^{-3}**.

- It can be seen that **ρ** and **ρ_c** are of the **same order of magnitude**, but since neither is known accurately, it is impossible at present to say which is the larger and hence predict the fate of the universe. The majority of cosmologists think that **ρ = ρ_c** and are advocates of what is termed a **flat universe**.

POSSIBLE FATES OF THE UNIVERSE

- The diagram below shows the three possible scenarios for the future of the universe.



- The following facts are now known to within 10% :
  - **The rate of expansion of the universe.**
  - **The mean separation of the galaxies.**

This means that the position where each line intersects the y-axis on the above graph is known to that degree of accuracy and that the gradient at that point is the same for all three graphs.

CLOSED UNIVERSE ($\rho > \rho_c$)

- In this outcome, the mean density (ρ) of the universe is so great that the gravitational forces will eventually cause **galaxies to slow down, stop and then be pulled back to produce a big crunch.**

OPEN UNIVERSE ($\rho < \rho_c$)

- In this outcome, the mean density (ρ) of the universe is too low and **each galaxy will reach a constant velocity and, at these velocities, all galaxies will continue to separate for ever.**

FLAT UNIVERSE ($\rho = \rho_c$)

- This is a boundary scenario which is unlikely, but most cosmologists believe in it. What it implies is that the **galaxies will gradually slow down, but never actually stop.**

- Note that, each possibility implies a different zero time for the big bang. When, in the future, the actual time at which the big bang occurred is accurately known, we will come to know the fate of the universe with some certainty.

- **HOMEWORK QUESTION**

1 The critical density of the universe can be shown to be given by the Equation :

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

(a) State **two assumptions** made in the derivation of this equation.

(b) Calculate the **critical density of the universe**, giving your answer in **hydrogen atoms per cubic metre**.

Hubble constant (H_0) = 1.6×10^{-18} s^{-1}.
Mass of a hydrogen atom = 1.7×10^{-27} kg.

(c) Theory suggests that the universe may have three possible fates, referred to as **open**, **flat** and **closed**.

Describe each of these and illustrate the evolution of the universe in each case, by a suitable sketch graph with **size on the y-axis** and **time on the x-axis**.

GOOD LUCK IN YOUR EXAMS

Remember, there is no substitute for solid hard study and revision, so pull out all the stops in these last weeks and do yourselves proud.

FXA and AF